

Fano and Dicke effects and spin polarization in a double Rashba-ring system side coupled to a quantum wire

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The electronic transport in a system of two quantum rings side-coupled to a quantum wire is studied via a single-band tunneling tight-binding Hamiltonian. We derived analytical expressions for the conductance and spin polarization when the rings are threaded by magnetic fluxes with Rashba spin-orbit interaction. We show that by using the Fano and Dicke effects this system can be used as an efficient spin-filter even for small spin orbit interaction and small values of magnetic flux. We compare the spin-dependent polarization of this design and the polarization obtained with one ring side coupled to a quantum ring. As a main result, we find better spin polarization capabilities as compared to the one ring design

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I. INTRODUCTION

Electronic transport through quantum rings structures has become the subject of active research during the last years. Interesting quantum interference phenomena have been predicted and measured in these mesoscopic systems in presence of a magnetic flux, such as the Aharonov-Bohm oscillations in the conductance, persistent currents^{1,2,3} and Fano antiresonances^{4,5}.

Recently, there has been much interest in understanding the manner in which the unique properties of nanostructures may be exploited in spintronic devices, which utilize the spin degree of freedom of the electron as the basis of their operation^{6,7,8,9,10,11}. A natural feature of these devices is the direct connection between their conductance and their quantum-mechanical transmission properties, which may allow their use as an all-electrical means for generating and detecting spin polarized distributions of carriers. For instance, recently Son et al.⁷ described how a spin filter may be realized in open-quantum dot system, by exploiting the Fano resonances that occur in their transmission. In a quantum dot in which the spin degeneracy of carrier is lifted, they showed that the Fano effect may be used as an effective means to generate spin polarization of transmitted carriers, and that electrical detection of the resulting polarization should be possible. This idea was extended to side attached quantum rings. In Ref.(12) Shelykh et. al. analyze the conductance of the Aharonov-Bohm (AB), one-dimensional quantum ring touching a quantum wire. They found that the period of the AB oscillations strongly depends on the chemical potential and the Rashba coupling parameter. The dependence of the conductance on the carrier's energy reveals the Fano antiresonances. On the other hand, Bruder et. al.¹³ introduce a spin filter based on spin-resolved Fano resonances due to spin-split levels in a quantum ring side coupled to a quantum wire. Spin-orbit coupling inside the quantum ring, together with external magnetic fields, induces spin splitting, and the Fano resonances due to the spin-split levels result in per-

fect or considerable suppression of the transport of either spin direction. They found that the Coulomb interaction in the quantum ring enhances the spin-filter operation by widening the separation between dips in the conductance for different spins and by allowing perfect blocking for one spin direction and perfect transmission for the other.

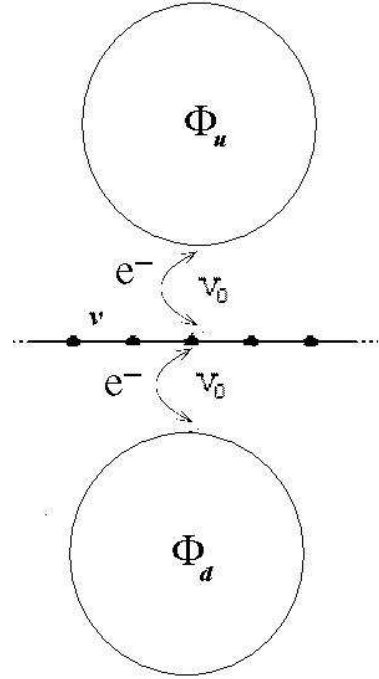


FIG. 1: Schematic view of the two quantum ring attached to quantum wire.

In this paper we study the two ring side-coupled to a quantum wire in presence of magnetic flux and Rashba spin-orbit interaction, as shown schematically in Fig. 1. In a previous paper (ref 14) we investigate the conductance and the persistent current of two mesoscopic quantum ring attached to a perfect quantum wire in presence

of a magnetic field. We show that the system develops an oscillating band with resonances (perfect transmission) and antiresonances (perfect reflection). In addition, we found persistent current magnification in the rings due to the Dicke effect in the rings when the magnetic flux difference is an integer number of the quantum of flux. The Dicke effect in optics takes place in the spontaneous emission of a pair of atoms radiating a photon with a wave length much larger than the separation between them.¹⁵ The luminescence spectrum is characterized by a narrow and a broad peak, associated with long and short-lived states, respectively. Now, we show that by using the Fano and Dicke effects this system can be used as an efficient spin-filter even for small spin orbit interaction and small values of magnetic flux. We find that the spin-polarization dependence for this system is much more sensitive to magnetic flux and spin-orbit interaction than the case with only one ring side-coupled to the quantum wire.

II. MODEL

In the presence of Rashba the spin-orbit coupling and magnetic flux Φ_{AB} , the Hamiltonian for an isolated one-dimensional rings reads¹⁶,

$$H = \hbar\Omega \left[\left(-i\frac{\partial}{\partial\varphi} - \frac{\Phi_{AB}}{\Phi_0} + \frac{\omega_{so}}{\Omega}\sigma_r(\varphi) \right)^2 - \frac{\omega_{so}^2}{4\Omega^2} \right] \quad (1)$$

where,

$$\sigma_r(\varphi) = \cos(\varphi)\sigma_x + \sin(\varphi)\sigma_y$$

where σ_x , σ_y and σ_z are the Pauli matrices. The parameter $\hbar\Omega = \frac{\hbar^2}{2ma^2}$ and $\omega_{so} = \frac{\alpha_{so}}{\hbar a}$ is the frequency associated to the SO coupling. The spin-orbit coupling constant α_{so} depends implicitly on the strength of the surface electric field¹⁷. The energy spectrum of the above Hamiltonian is given by,

$$\varepsilon_{\mu n} = \hbar\Omega \left[\left(n - \phi_{AB} + \frac{1}{2} - \mu \frac{1}{2\cos\theta} \right)^2 - \frac{1}{4}\tan^2\theta \right] \quad (2)$$

where $\theta = -\arctan(\omega_{so}/\Omega)$ and $\phi_{AB} = \frac{\Phi_{AB}}{\Phi_0}$, the Aharonov-Bohm phase.

The eigenstates are given by the following wave functions,

$$\Psi_n^+(\varphi) = e^{in\varphi} \begin{pmatrix} \cos(\frac{\theta}{2}) \\ e^{i\varphi} \sin(\frac{\theta}{2}) \end{pmatrix}$$

$$\Psi_n^-(\varphi) = e^{in\varphi} \begin{pmatrix} \sin(\frac{\theta}{2}) \\ -e^{i\varphi} \cos(\frac{\theta}{2}) \end{pmatrix}$$

The second quantization form of the quantum wire-quantum ring device with a magnetic flux and spin-orbit interaction can be written as,

$$H_T = \sum_{i\mu} \varepsilon_i c_{\mu,i}^\dagger c_{\mu,i} + v \sum_{\langle ij \rangle \mu} \left(c_{\mu,i}^\dagger c_{\mu,j} + h.c \right) + \sum_{\alpha,n,\mu} \varepsilon_{\mu,n}^\alpha d_{\mu,n}^{\alpha\dagger} d_{\mu,n}^\alpha + V_0 \sum_{\mu,n,\alpha} (d_{\mu,n}^{\alpha\dagger} c_{\mu 0} + h.c) \quad (3)$$

The operator $c_{j\mu}^\dagger$ creates an electron in the site j of the wire and with spin index μ , $d_{n\mu}^{\alpha\dagger}$ creates an electron in the level n of the ring α and with spin index μ . The wire site-energy is assumed equal to zero and the hopping energies for wire and rings are taken to be equal to v , whereas V_0 couples both systems.

Within the described model the conductance can be calculated by means of a Dyson equation for the Green's function.

$$G_{\mu 0}^\alpha = \frac{i}{2v\sqrt{1 - \frac{\omega^2}{4v^2}}} \frac{1}{1 - i\gamma \sum_\beta A_\mu^\beta(\omega)} \quad (4)$$

where $\gamma = \frac{V_0^2}{2v\sqrt{1 - \frac{\omega^2}{4v^2}}}$ and

$$A_\mu^\alpha(\omega) = \sum_{n=-\infty}^{\infty} g_{n\mu}^\alpha = \sum_{n=-\infty}^{\infty} \frac{1}{\omega - \varepsilon_{\mu n}^\alpha} \quad (5)$$

and,

$$g_{n\mu}^\alpha = \frac{1}{\omega - \varepsilon_{\mu n}^\alpha}. \quad (6)$$

Where $g_{n\mu}^\alpha$ is the Green's function of the isolated ring α .

The conductance of the system can be calculated using the Landauer formula.

$$\mathcal{G}_\mu = \frac{e^2}{h} T_\mu(\omega = E_F) \quad (7)$$

where T_μ is the probability transmission. In the linear response approach it can be written in term of the Green's function of the contact by:

$$T_\mu(\omega) = \Gamma(\omega) \Im m [G_{\mu 0}^\alpha(\omega)] = \frac{1}{1 + \gamma^2 \left[\sum_\beta A_\mu^\beta(\omega) \right]^2}, \quad (8)$$

where $\Gamma(\omega) = 2v\sqrt{1 - \frac{\omega^2}{4v^2}}$.

Following ref. 7 we introduce the weighted spin polarization as

$$P_\mu = \frac{|T_+ - T_-|}{|T_+ + T_-|} T_\mu, \quad \mu = \pm. \quad (9)$$

Notice that this definition takes into account not only the relative fraction of one of the spins, but also the contribution of those spins to the electric current. In other words, we will require that not only the first term of the right-hand side of (9) to be of order of unity, but also the transmission probability T_μ .

III. RESULTS

In what follow we present results for the conductance and spin polarization for a double ring system of radius $a = 120nm$, coupled each other through a quantum-wire. For this radius the energy $\hbar\Omega = 40\mu eV$. We consider only energies near of the center of the band therefore we consider the tunneling coupling as a constant. Then we set the tunneling coupling $\gamma = 16\mu eV$.

By using the results given in ref.[18] $A_\mu^\beta(\omega)$ can be evaluated analytically,

$$A_\mu^\alpha(\omega) = \frac{2\pi^2}{\hbar\Omega z} \frac{\sin(z)}{\cos(2\pi\phi_\mu^\alpha) - \cos(z)}$$

$$z = \pi \left(\frac{4\omega}{\hbar\Omega} + \frac{\omega_{so}^2}{\Omega^2} \right)^{1/2}$$

where, $\phi_\mu^\alpha = \phi_{AB}^\alpha + \frac{1}{2} - \mu \frac{1}{2\cos\theta}$, is the net phase for the α -ring. Then, we can obtain an analytical expression for the conductance,

$$\mathcal{G}_\mu(\omega) = \frac{e^2}{h} \frac{[(\cos(2\pi\phi_\mu^u) - \cos(z)) (\cos(2\pi\phi_\mu^d) - \cos(z))]^2}{[(\cos(2\pi\phi_\mu^u) - \cos(z)) (\cos(2\pi\phi_\mu^d) - \cos(z))]^2 + \beta^2 [\cos(2\pi\phi_\mu^u) + \cos(2\pi\phi_\mu^d) - 2\cos(z)]^2}. \quad (10)$$

with $\beta = (\gamma 2\pi^2 / \hbar\Omega) (\sin z / z)$

An interesting situation appears when the energy spectrum of both rings becomes degenerated. This occurs when the magnetic fluxes threading the rings are equals ($\phi_{AB}^u = \phi_{AB}^d = \phi_{AB}$). For this case we obtain,

$$\mathcal{G}_\mu = \frac{e^2}{h} \frac{(\cos(2\pi\phi_\mu) - \cos(z))^2}{(\cos(2\pi\phi_\mu) - \cos(z))^2 + 4\beta^2}.$$

The spin-dependent conductance vanishes when $\cos(2\pi\phi_\mu) - \cos(z) = 0$, i.e., when $E_F = \varepsilon_\mu^\alpha$. The zeroes in the conductance (Fano antiresonances) represent exactly the superposition of the spectrum of isolated rings. In fact, the conductance can be written as superposition of symmetric Fano line-shapes

$$\mathcal{G}_\mu = \frac{e^2}{h} \frac{(\epsilon_\mu + q)^2}{\epsilon_\mu^2 + 1}.$$

where, $\epsilon_\mu = (\cos(2\pi\phi_\mu) - \cos(z)) / 2\beta$ is the detuning parameter and q is the Fano parameter, in this case $q = 0$.

Figure 2 displays the spin-dependent linear conductance (upper layers) and spin polarization (lower layers) versus the Fermi energy for the symmetric case with $\phi_{AB} = 0.25$ and a spin-orbit coupling $\alpha_{so} = 0.5 \times 10^{-11} eVm$, $\phi_{AB}^u = \phi_{AB}^d = 0.25$.

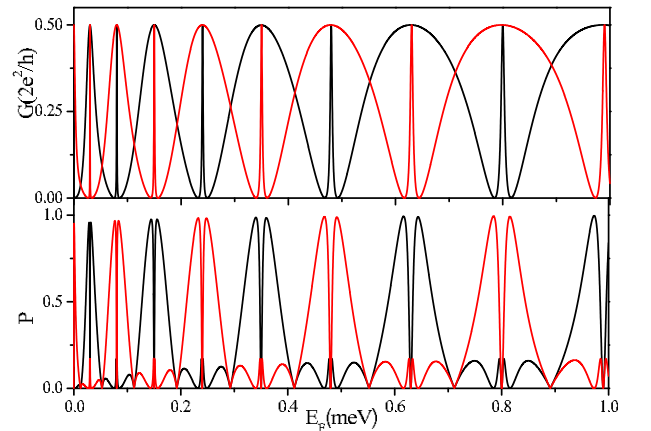


FIG. 2: Spin-dependent conductance(upper layer) and spin polarization (lower layer) as a function of Fermi energy, (color online, black line $\mu = +$, red line ($\mu = -$)) for $\alpha_{so} = 0.5 \times 10^{-11} eVm$, $\phi_{AB}^u = \phi_{AB}^d = 0.25$.

$10^{-11}eVm$. The energy spectrum consists of a superposition of quasi-bound states reminiscent of the corresponding localized spectrum of the isolated rings. As expected from the analytical expression (Eq. 10) the linear conductance displays a series of resonances and Fano antiresonances as a function of the Fermi energy. On the other hand, for given set of parameters the system shows zones of high polarization due to the splitting of the spin energy states.

Now we analyze the asymmetric case, i.e $\phi_{AB}^u \neq \phi_{AB}^d$. Figure 3 displays the spin-dependent linear conductance (upper layers) and spin polarization (lower layers) versus the Fermi energy for a spin-orbit coupling $\alpha_{so} = 0.5 \times 10^{-11}eVm$ and parameters of magnetic flux given by $\phi_{AB}^u = 0.2, \phi_{AB}^d = 0.3$. Newly, the zeroes in the conductance represent exactly the superposition of the spectrum of each isolated ring $\varepsilon_{\mu n}^\alpha$. In fact, now the conductance vanishes when, $\cos(2\pi\phi_\mu^u) - \cos(z) = 0$ or $\cos(2\pi\phi_\mu^d) - \cos(z) = 0$, i.e when $E_F = \varepsilon_\mu^\alpha$. Notice that due to the difference between both fluxes new resonances in the conductance appear. This also affects the structure of the polarization.

We note that when there is a magnetic flux difference $\delta\phi_{AB} = \phi_{AB}^u - \phi_{AB}^d$ high spin polarization can obtain even for small values of the spin-orbit coupling. In fact, for small values spin-orbit coupling by adjusting the magnetic flux difference $\delta\phi_{AB}$ maxima of polarization are reached. We analyze in detail this situation. The maxima of the conductance are obtained when $\sin z = 0$ or when $(\cos(2\pi\phi_\mu^u) + \cos(2\pi\phi_\mu^d) - 2\cos(z)) = 0$. The first condition is spin-independent and it is not interesting in this case. The second condition is spin-dependent and for small magnetic flux differ-

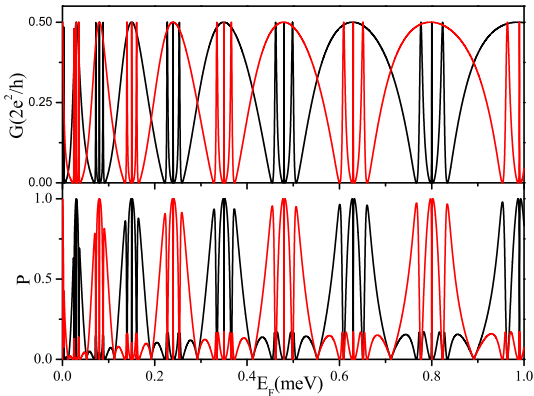


FIG. 3: Spin-dependent conductance (upper layer) and spin polarization (lower layer) as a function of Fermi energy, (color online, black line $\mu = +$, red line $\mu = -$) for $\alpha_{so} = 0.5 \times 10^{-11}eVm, \phi_{AB}^u = 0.3$ and $\phi_{AB}^d = 0.2$

ence can be written as, $\cos\left[2\pi\left(\frac{\phi_\mu^u + \phi_\mu^d}{2}\right)\right] - \cos(z) \approx 0$. This occurs for the energies given by $\tilde{\varepsilon}_{\mu n} = \hbar\Omega\left[\left(n - \tilde{\phi}_\mu + \frac{1}{2} - \mu\frac{1}{2\cos\theta}\right)^2 - \frac{1}{4}\tan^2\theta\right]$, where $\tilde{\phi}_\mu = \frac{\phi_\mu^u + \phi_\mu^d}{2}$ i.e the position of the maxima of the conductance corresponding to the spectrum of an effective ring with phase $\tilde{\phi}_\mu$. Therefore the condition for the maxima of polarization are given when the minima of the conductance for one spin-state coincide with the maxima of the conductance of the opposite spin (or viceversa), that is $\tilde{\varepsilon}_{\mu n} = \varepsilon_{\mu n+1}^\alpha$, then $\delta\phi_{AB} = (1 - \cos\theta) / \cos\theta \approx \frac{1}{2}(\omega_{so}/\Omega)^2$. Then, for a given spin-orbit coupling by adjusting the magnetic flux difference between the upper and lower rings, the maxima of the spin polarization are reached. Fig.4 displays the spin dependent conductance (upper layer) and the spin polarization (lower layer) for $\tilde{\phi}_{AB} = 0.25, \alpha_{so} = 5 \times 10^{-12}eVm$ and $\delta\phi_{AB} = 0.004988$. The conductance shows broad and sharp peaks and the spin polarization shows a series peaks of maximum of polarization. Fig.5 displays a zoom of the conductance (right panel) and the polarization (left panel) as a function of the Fermi energy. Clearly the sharp peaks and Fano antiresonances for the two spin states are shifted given origin to the peaks of maximum of polarization. For comparison we plot the corresponding conductance and polarization for one ring for the same values of the magnetic flux and spin orbit coupling (Fig. 6). For these parameter the spin polarization of one ring is very low for both spin states. The inset in Fig.6 (lower panel) shows a zoom of the spin polarization.

For small values of magnetic flux difference $\delta\phi_{AB}$ the conductance of the two ring system can be written approximately as a superposition of a broad Fano line shape

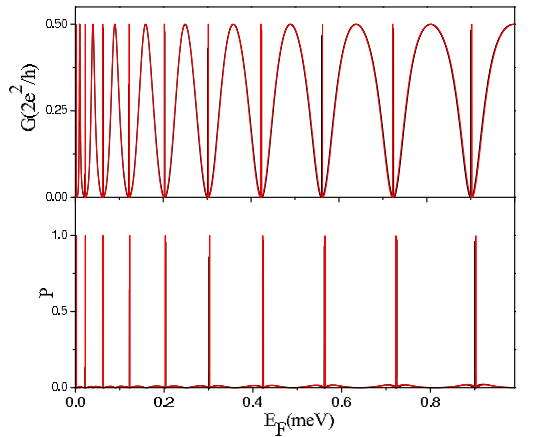


FIG. 4: Spin-dependent conductance (upper layer) and spin polarization (lower layer) as a function of Fermi energy, (color online, black line $\mu = +$, red line $\mu = -$) for $\alpha_{so} = 0.5 \times 10^{-12}eVm, \tilde{\phi}_{AB} = 0.25$ and $\delta\phi_{AB} = 0.004988$.

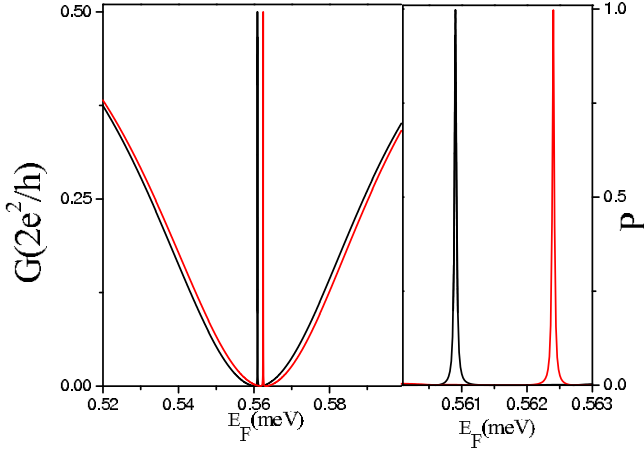


FIG. 5: Spin-dependent conductance (left panel) and spin polarization (right panel) as a function of Fermi energy, (color online, black line $\mu = +$, red line $\mu = -$) for $\alpha_{so} = 0.5 \times 10^{-12} \text{ eVm}$, $\tilde{\phi}_{AB} = 0.25$ and $\delta\phi_{AB} = 0.004988$.

and a narrow Breit-Wigner line shape. This is,

$$\mathcal{G}_\mu \approx \frac{e^2}{h} \left[\frac{(\epsilon_\mu + q)^2}{\epsilon_\mu^2 + 1} + \frac{\eta_\mu^2}{x_\mu^2 + \eta_\mu^2} \right]. \quad (11)$$

where the width $\eta_\mu = (\sin 2\pi\tilde{\phi}_\mu \sin 2\pi\delta\phi_{AB})^2 / (2\gamma\beta)$ and $x_\mu = 2\beta\epsilon_\mu$. As we discuss in a previous paper¹⁴, this expression clearly shows the superposition of short and long living states developed in the rings. The apparition of quasi-bound states in the spectrum of the system is a consequence of the mixing of the levels of both rings which are coupled indirectly through the continuum of states in the wire. A similar effect was discussed recently in a system with a ring coupled to a reservoir by Wunsch et al. in ref.[18]. They relate this kind of collective states with the Dicke effect in optics. The Dicke effect in

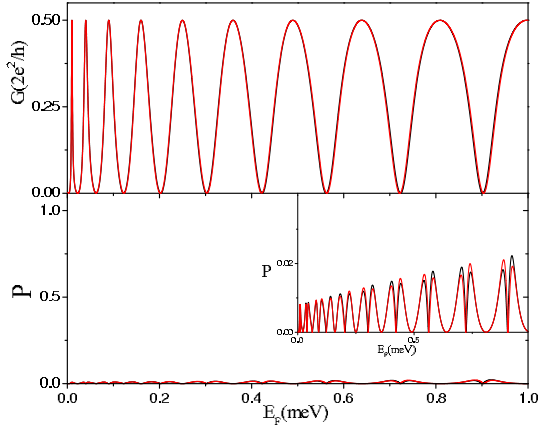


FIG. 6: One ring spin-dependent conductance (upper layer) and spin polarization (lower layer) as a function of Fermi energy, (color online, black line $\mu = +$, red line $\mu = -$) for $\alpha_{SO} = 0.5 \times 10^{-11} \text{ eVm}$, $\phi_{AB} = 0.25$.

optics takes place in the spontaneous emission of a pair of atoms radiating a photon with a wave length much larger than the separation between them.¹⁵ The luminescence spectrum is characterized by a narrow and a broad peak, associated with long and short-lived states, respectively. This feature allows to obtain high spin polarization even for small spin-orbit coupling by adjusting the magnetic flux difference $\delta\phi_{AB}$. High spin polarization holds even for small values of the magnetic flux. For instance the Fig. 7 displays the conductance and spin polarization as a function of the Fermi energy for $\tilde{\phi}_{AB} = 0.01$, $\alpha_{so} = 5 \times 10^{-12} \text{ eVm}$ and $\delta\phi_{AB} = 0.004988$. The spin-polarization shows sharp peaks for the two spin states. As a comparison with a single ring side-coupled to a quantum wire, the system composed by two rings allows us to obtain high spin polarization even for small spin-orbit interaction and small magnetic fluxes, keeping a small difference for these fluxes.

IV. SUMMARY

We have investigated the spin dependent conductance and spin polarization in a system of two side quantum rings attached to a quantum wire in the presence of magnetic fluxes threading the rings and Rashba spin-orbit interaction. We show that by using the Fano and Dicke effects this system can be used as an efficient spin-filter. We compare the spin-dependent polarization of this design and the polarization obtained with one ring side coupled to a quantum ring. As a main result, we find better spin polarization capabilities as compared to the one ring design. We find that the spin-polarization dependence for this system is much more sensitive to magnetic flux and spin-orbit interaction than the case with only one ring side-coupled to the quantum wire. This behavior is interesting from theoretical point of view, but also by its

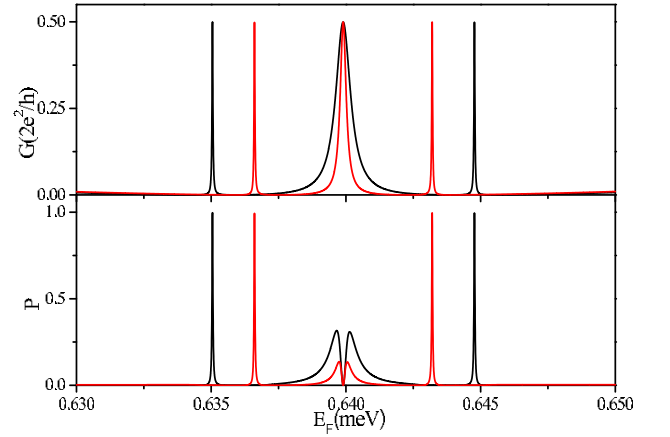


FIG. 7: Spin-dependent conductance (upper layer) and spin polarization (lower layer) as a function of Fermi energy, (color online, black line $\mu = +$, red line $\mu = -$) for $\alpha_{SO} = 0.5 \times 10^{-12} \text{ eVm}$, $\tilde{\phi}_{AB} = 0.01$ and $\delta\phi_{AB} = 0.004988$.

potential technological application.

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- ¹ V. Chandrasekhar, R.A. Webb, M.J. Brady, M.B. Ketchen, W.J. Gallagher and A. Kleinsasser, Phys. Rev. Lett. **67** 3578 (1991).
 - ² D. Mailly, C. Chapelier and A. Benoit, Phys. Rev Lett. **70** 2020 (1993).
 - ³ U. F. Keyser, C. Fühner, S. Borck and R. J. Haug, Semm. Sc. and Tech. **17**, L22 (2002).
 - ⁴ Jorge L. D'Amato, Horario M. Pastawski, and Juan F. Weisz, Phys. Rev. B **39**, 3554 (1989).
 - ⁵ P. A. Orellana, M. L. Ladrón de Guevara, M. Pacheco, and A. Latgé, Phys. Rev. B **68**, 195321 (2003).
 - ⁶ Datta S and Das B Appl. Phys. Lett. **56** 665 (1990)
 - ⁷ Song J F, Ochiai Y and Bird J P Appl. Phys. Lett. **82** 4561 (2003)
 - ⁸ Folk J A, R. M. Potok, Marcus C M and Umansky V *Science* **299** 679 (2003)
 - ⁹ Mireles F and Kirczenow G Phys. Rev. B **64** 024426 (2001)
 - ¹⁰ Mireles F, Ulloa S E, Rojas F and Cota E Appl. Phys. Lett. **88** 093118 (2006)
 - ¹¹ Berciu M and Janko B Phys. Rev. Lett. *Lett.***90** 246804 (2003)
 - ¹² I. A. Shelykh, N. G. Galkin, and N. T. Bagraev Phys. Rev. B **74**, 165331 (2006).
 - ¹³ Minchul Lee and C. Bruder Phys. Rev. B **73**, 085315 (2006).
 - ¹⁴ P. A. Orellana, and M. Pacheco, Phys. Rev. B **71**, 235330 (2005).
 - ¹⁵ R. H. Dicke, Phys. Rev. **89**, 472 (1953).
 - ¹⁶ F. E. Meijer, A. F. Morpurgo, and T. M. Klapwijk, Phys. Rev. B **66**, 033107 (2002).
 - ¹⁷ J. Luo, H. Munekata, F. F. Fang and P. J. Stiles Phys. Rev. B **38**, 10142 - 10145 (1988)
 - ¹⁸ B. Wunsch, A. Chudnovskiy, Phys. Rev. B **68**, 245317 (2003).